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15MATDIP41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2021
Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix
$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$
by applying elementary row transformations. **(06 Marks)**

   b. Solve the system of equations by Gauss-elimination method:
   $x + y + z = 6$
   $x - y + 2z = 5$
   $3x + y + z = 8$ **(05 Marks)**

   c. Find all eigen values and eigen vectors of the matrix
   $$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$ **(05 Marks)**

OR

2 a. Find all eigen values and all eigen vectors of the matrix
   $$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$ **(06 Marks)**

   b. Solve by Gauss elimination method:
   $3x + y + 2z = 3$
   $2x - 3y - z = -3$
   $x + 2y + z = 4$ **(05 Marks)**

   c. Find the inverse of the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ using Cayley-Hamilton theorem. **(05 Marks)**

Module-2

3 a. Solve $(D^3 - 6D^2 + 11D - 6)y = 0$ **(06 Marks)**

   b. Solve $(D^2 + 6D + 9)y = 2e^{-3x}$ **(05 Marks)**

   c. Solve by the method of variation of parameters $(D^2 + 1)y = \tan x$. **(05 Marks)**

OR

4 a. Solve $(D^3 - 5D^2 + 8D - 4)y = 0$ **(06 Marks)**

   b. Solve $(D^2 - 4D + 3)y = \cos 2x$ **(05 Marks)**

   c. Solve by the method of undetermined coefficients $y'' - y' - 2y = 1 - 2x$. **(05 Marks)**

Module-3

5 a. Find Laplace transform of $\cos^3 at$. **(06 Marks)**

b. A periodic function of period 2a is defined by

$$f(t) = \begin{cases} E & \text{for } 0 \le t \le a \\ -E & \text{for } a \le t \le 2a \end{cases}$$ where E is a constant. Find $L\{f(t)\}$. **(05 Marks)**

c. Express the function $f(t) = \begin{cases} \cos t, & t \le \pi \\ \sin t, & t > \pi \end{cases}$ in terms of unit step function and hence find its

Laplace transform. **(05 Marks)**

OR

6 a. Find $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$ **(06 Marks)**

b. Find $L\{\sin t \sin 2t \sin 3t\}$ **(05 Marks)**

c. Express the function $f(t) = \begin{cases} t^2, & t \le 2 \\ 4t, & t > 2 \end{cases}$ in terms of unit step function and hence find its

Laplace transform. **(05 Marks)**

Module-4

7 a. Find $L^{-1}\left\{\dfrac{2s+3}{s^3 - 6s^2 + 11s - 6}\right\}$ **(06 Marks)**

b. Find $L^{-1}\left\{\cot^{-1}\left(\dfrac{s}{a}\right)\right\}$ **(05 Marks)**

c. Using Laplace transform method, solve the initial value problem $y'' + 5y' + 6y = 5e^{2t}$, given that $y(0) = 2$ and $y'(0) = 1$. **(05 Marks)**

OR

8 a. Find $L^{-1}\left\{\dfrac{s+2}{s^2 - 4s + 13}\right\}$ **(06 Marks)**

b. Find $L^{-1}\left\{\log\left(\dfrac{s^2+1}{s(s+1)}\right)\right\}$ **(05 Marks)**

c. Using Laplace transforms, solve the initial value problem $y' + y = \sin t$, given that $y(0) = 0$. **(05 Marks)**

Module-5

9 a. For any two events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **(06 Marks)**

b. If A and B are any two events with $P(A) = \dfrac{1}{2}$, $P(B) = \dfrac{1}{3}$ and $P(A \cap B) = \dfrac{1}{4}$, find P(A/B), P(B/A), $P(\overline{A}/\overline{B})$ and $P(\overline{B}/\overline{A})$ **(05 Marks)**

c. From 6 positive and 8 negative numbers, 4 numbers are selected at random and are multiplied. What is the probability that the product is positive? **(05 Marks)**

OR

10 a. State and prove Baye's theorem. **(06 Marks)**

b. A book shelf contains 20 books of which 12 are on electronics and 8 are on mathematics. If 3 books are selected at random, find the probability that all the 3 books are on the same subject. **(05 Marks)**

c. The machines A, B, C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. If an item is selected at random is found to be defective, then determine the probability that the item was manufactured by machine A. **(05 Marks)**